Theta-3 is connected

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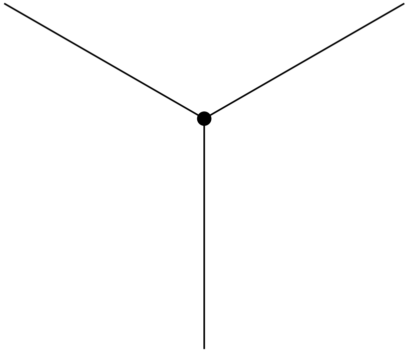
25th Canadian Conference on Computational Geometry

θ-graphs

- Partition plane into cones
- Add edge to 'closest' vertex in each cone



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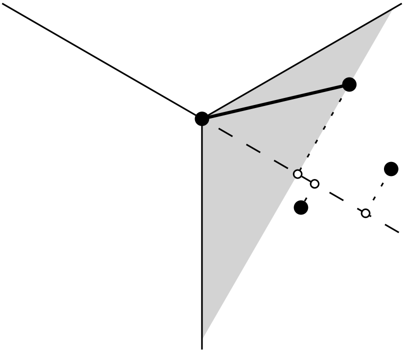
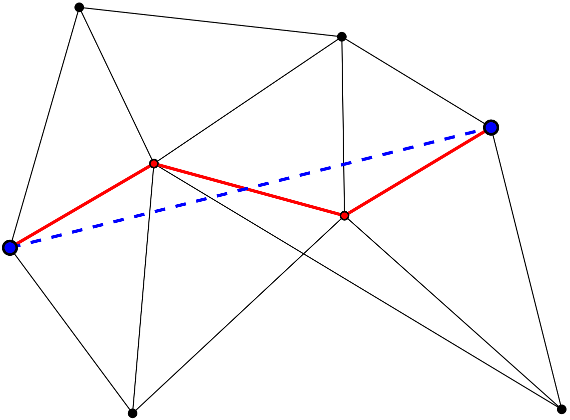


Geometric Spanners

- Graphs with short detours between vertices
- For every u and w, there is a path with length $\leq t \cdot |uw|$



Previous Work

Clarkson	1987	θ-graphs with > 8 cones are spanners
Keil	1988	
Ruppert & Seidel	1991	θ-graphs with > 6 cones are spanners

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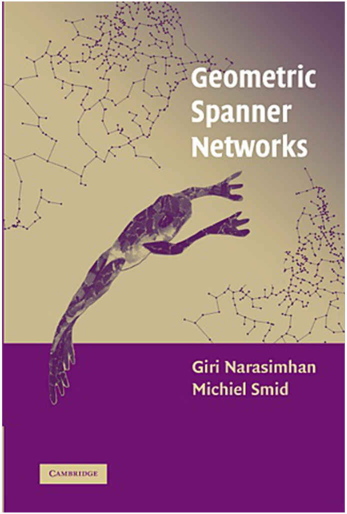

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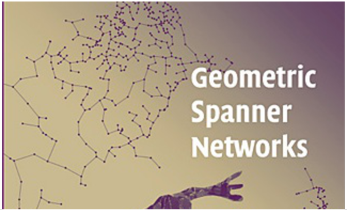
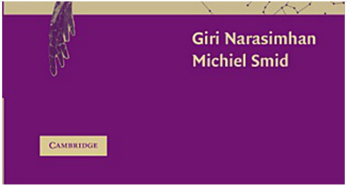


Exercises

4.1. What can you prove about the stretch factor of $\Theta(S, \kappa)$ if $\kappa \leq 8$? In particular, is $\Theta(S, \kappa)$ connected for such values of κ?



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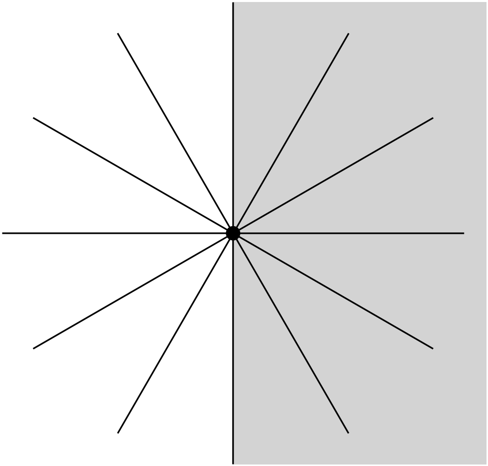
El Molla	2009	θ_2 and θ_3 are **not** spanners
Bonichon *et al.*	2010	θ_6 is a planar 2-spanner
Barba *et al.* Bose *et al.*	2013	θ_4 and θ_5 are spanners

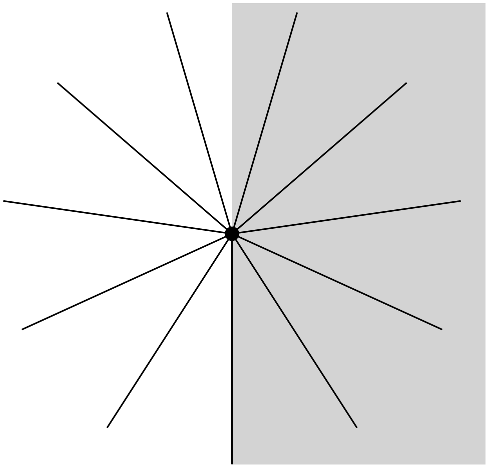
Connectedness

- Even θ-graphs



Connectedness

- Odd θ-graphs



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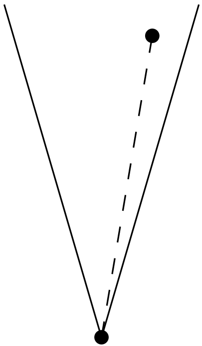

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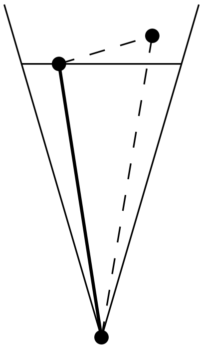
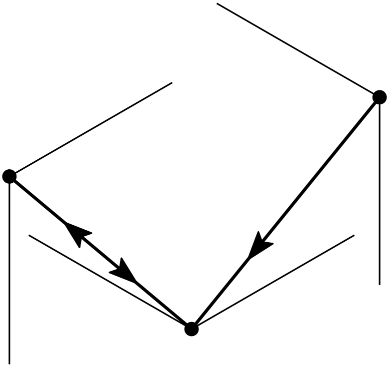


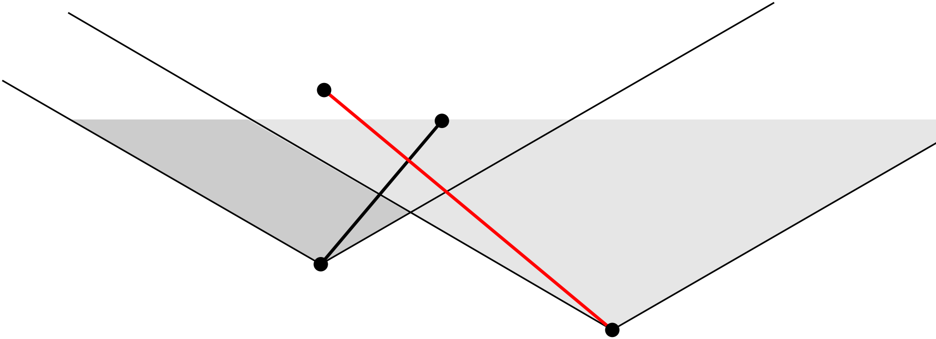
Connectedness

- Theta-routing does not work in θ_3



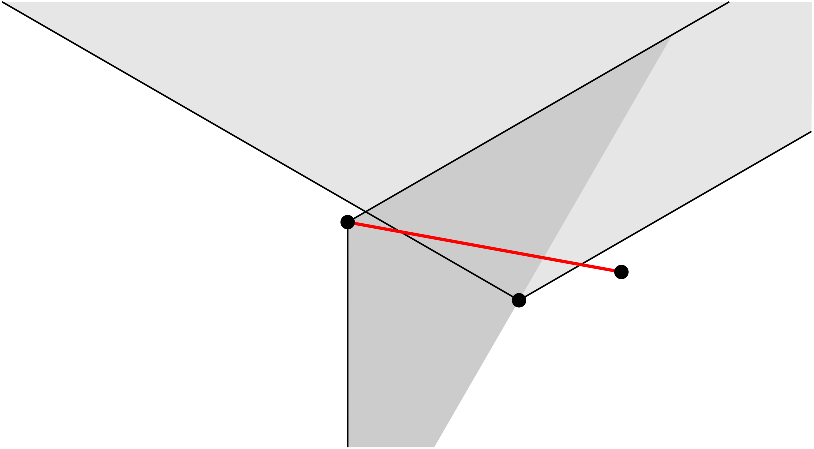
Properties

- Edges in the same cone cannot cross



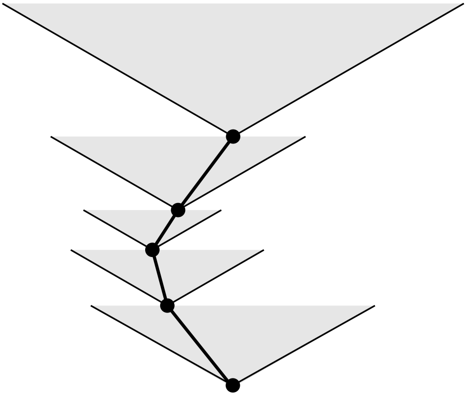
Properties

- Edges in the same cone cannot cross
- Edges cannot cross empty cones



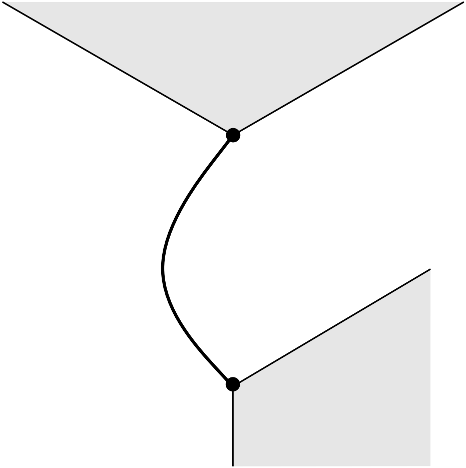
Paths

- Unique up-path from each vertex



Paths

- Paths can form *barriers*



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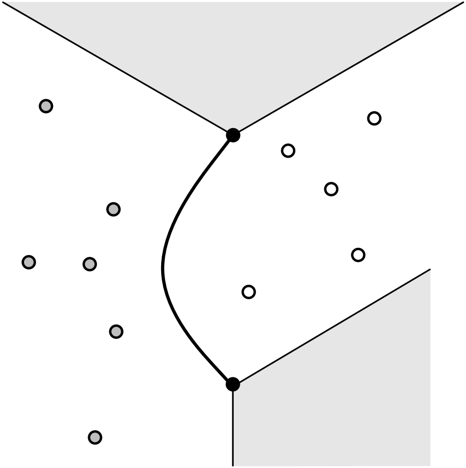
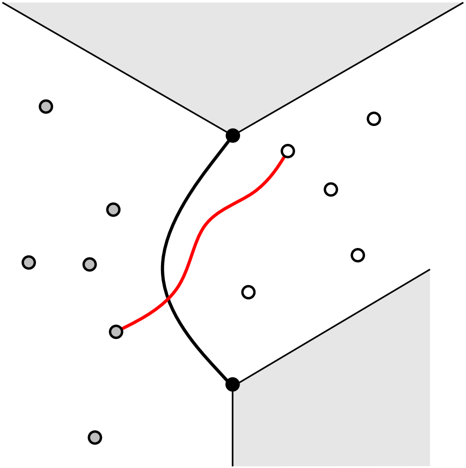


Paths

- Up-paths cannot cross up-barriers



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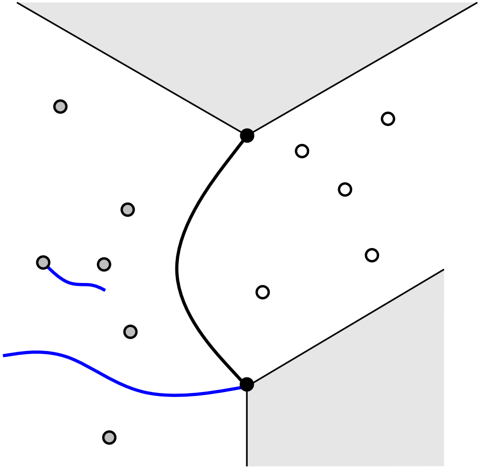

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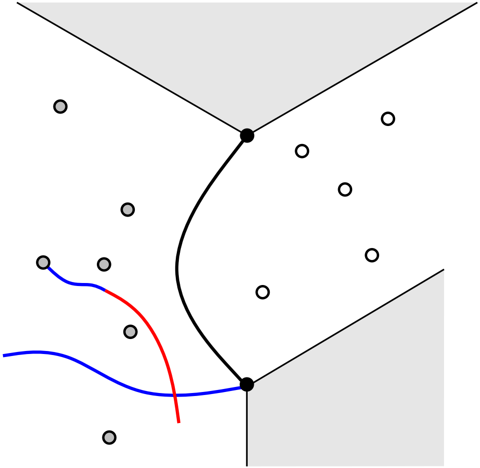

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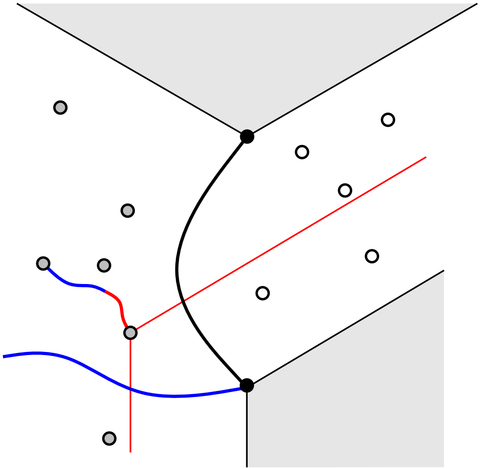
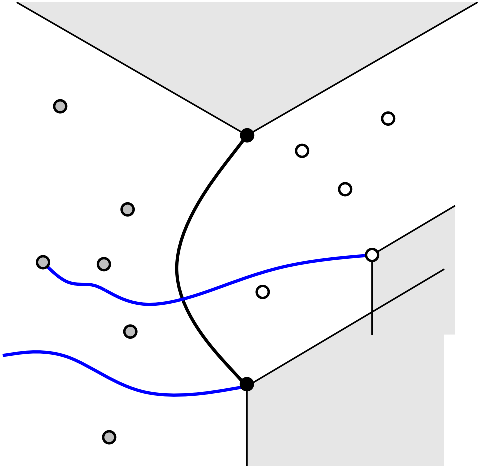


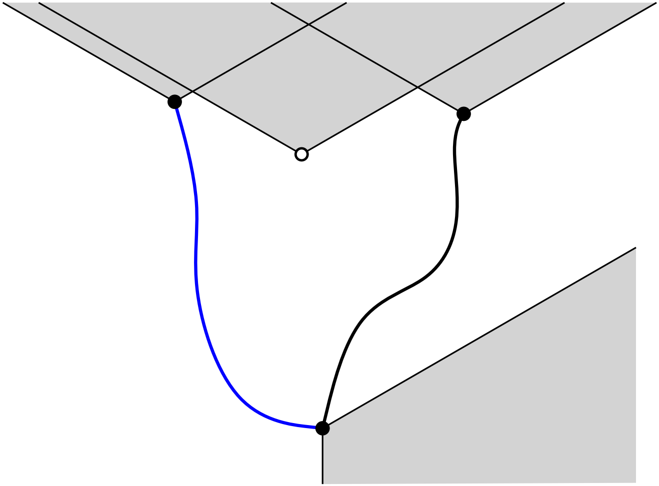
Paths

- Up-paths cannot cross up-barriers
- Other paths can be *forced* to cross up-barriers



Lemma

- Special configuration of up-sinks



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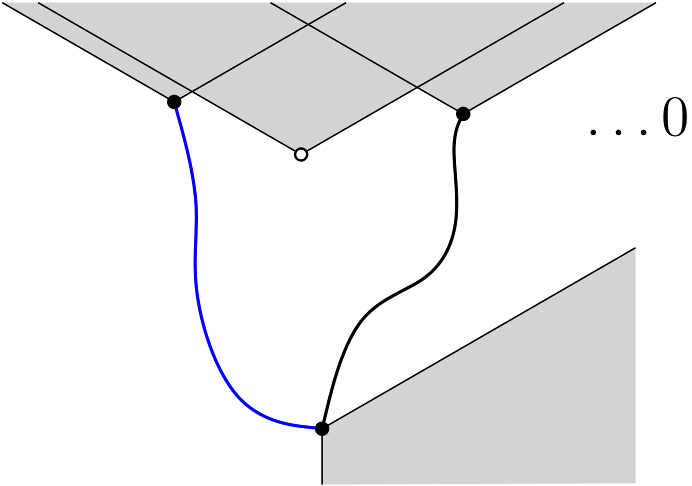

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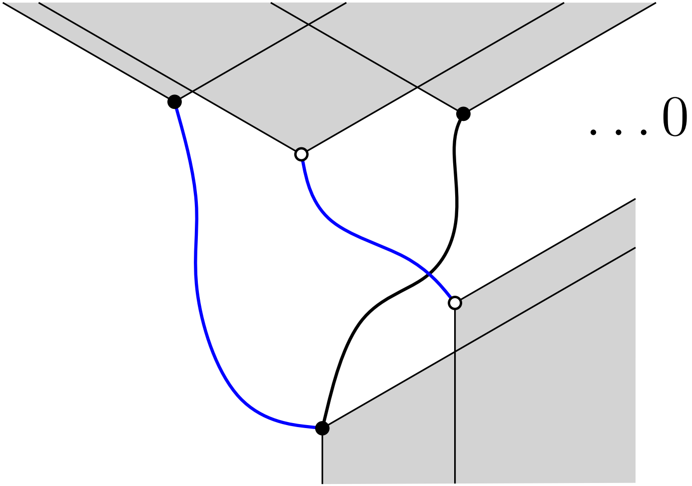

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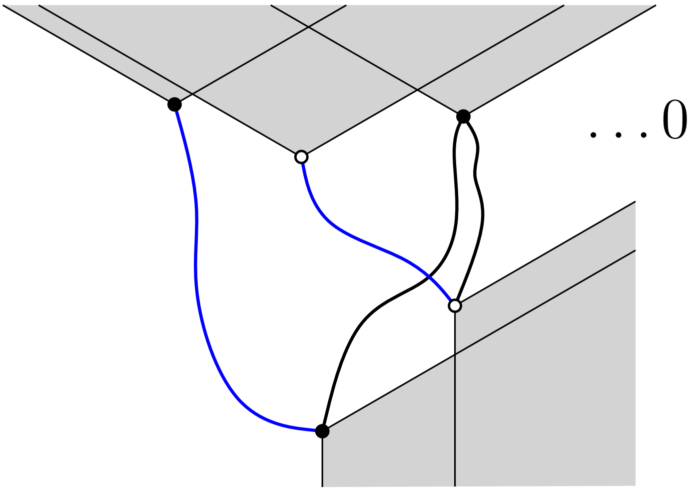

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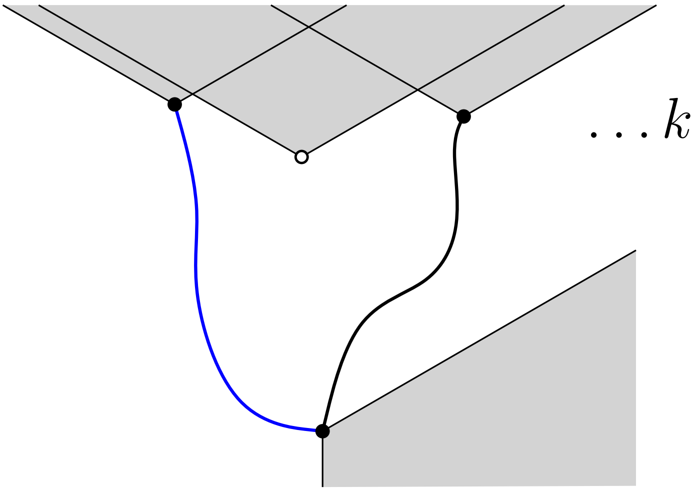

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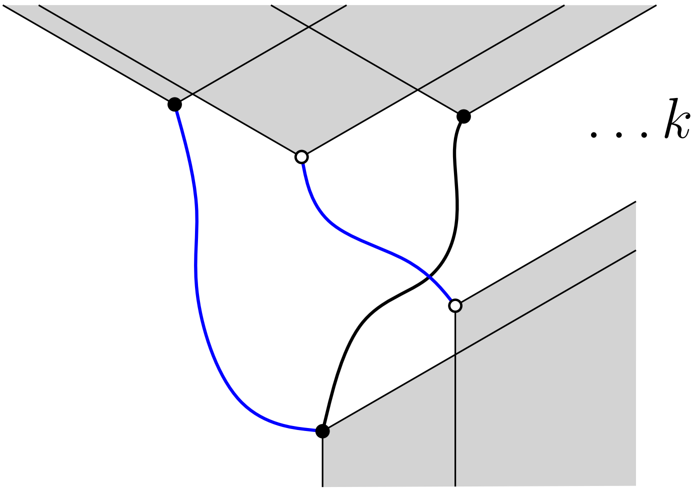

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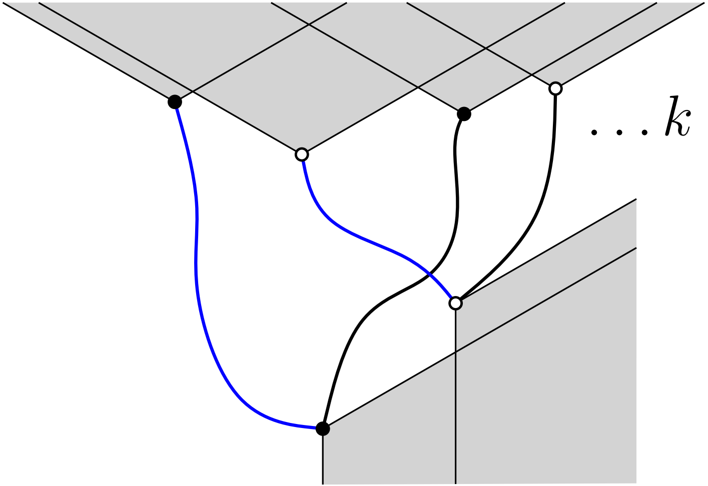

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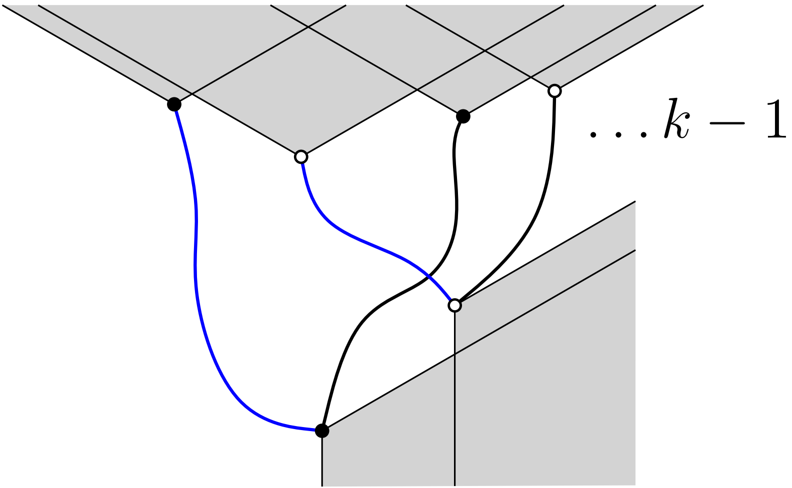

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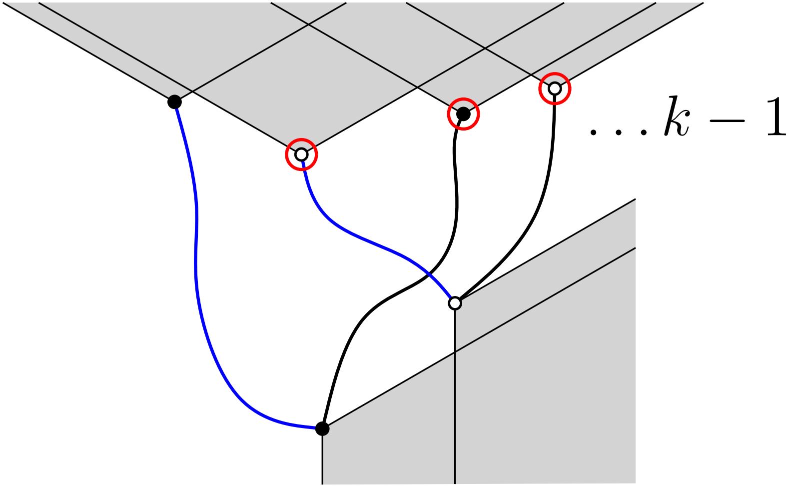

Proof

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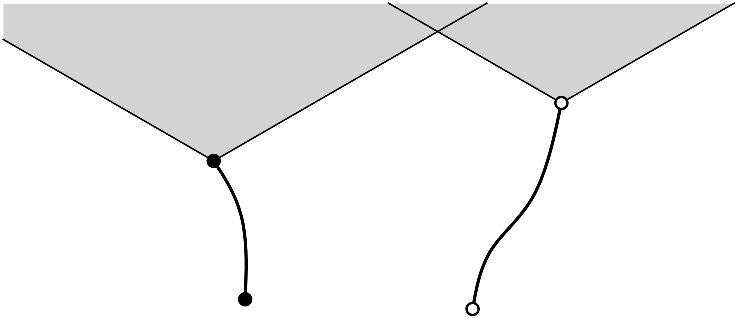

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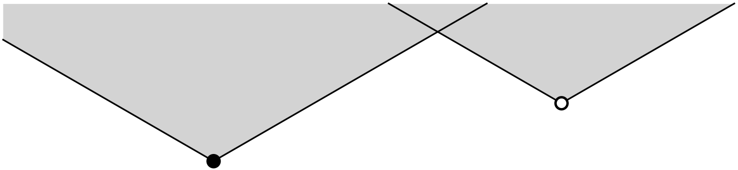

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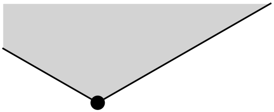

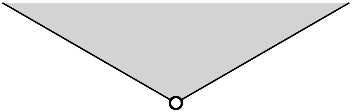

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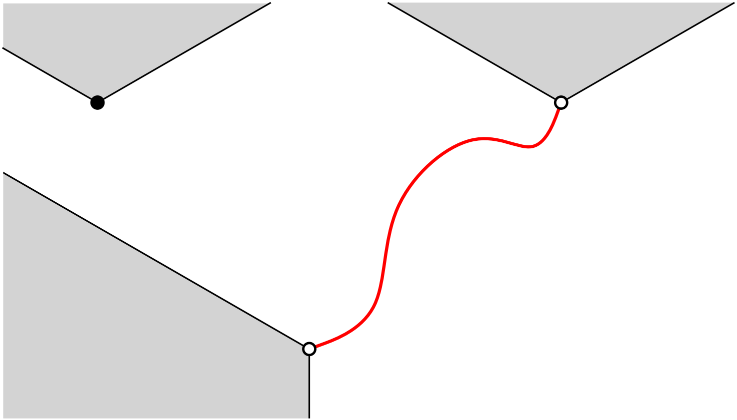

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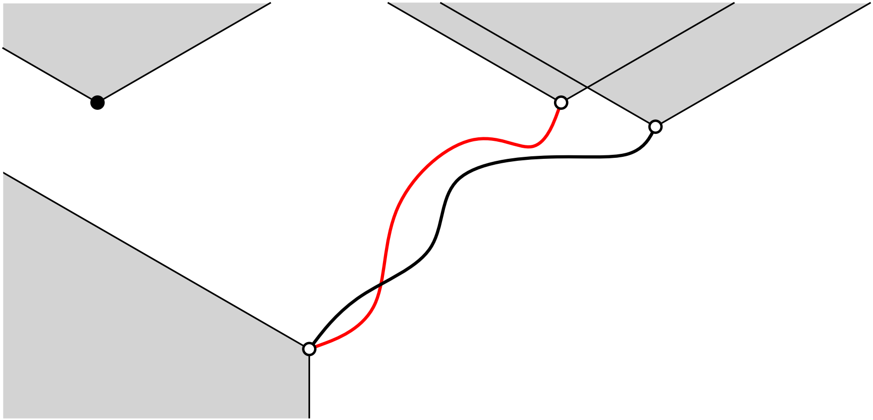

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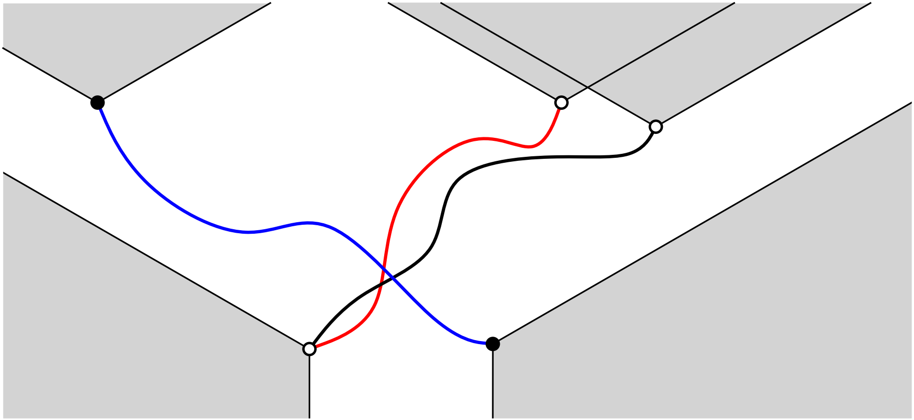

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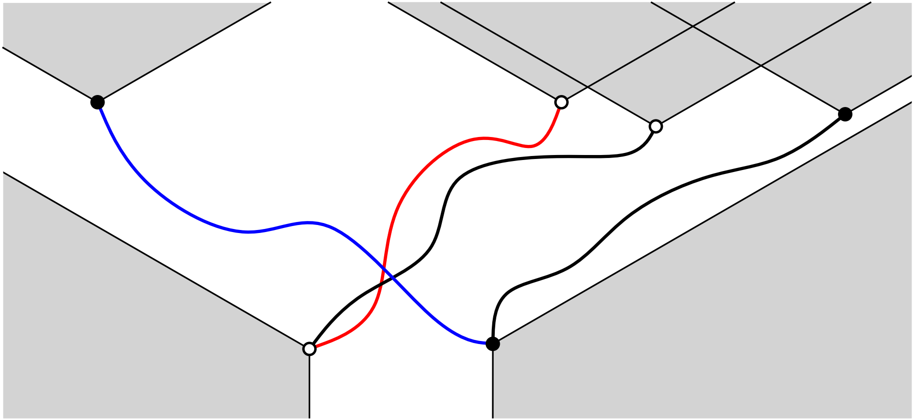

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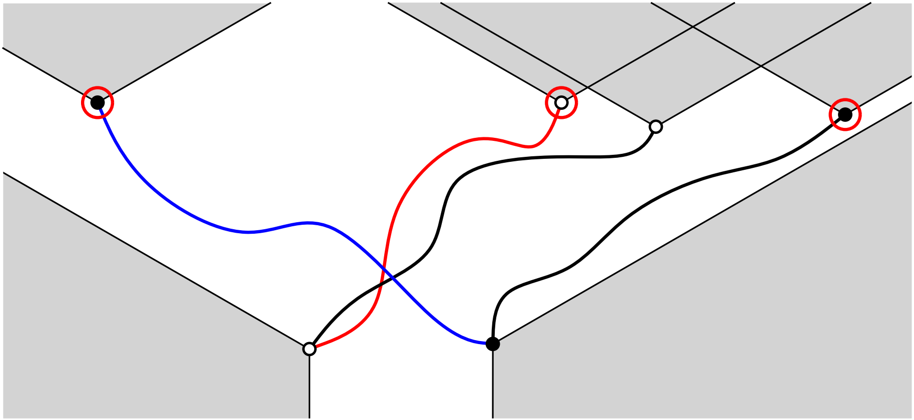

Conclusion

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- Properties hold for Yao_3 as well
  $\Rightarrow$ Yao_3 is connected

Future work

Exercises

4.1. What can you prove about the stretch factor of $\Theta(S, \kappa)$ if $\kappa \leq 8$? In particular, is $\Theta(S, \kappa)$ connected for such values of κ?

4.2. Algorithm Θ-Walk(p, q) in Section 4.1.1 computes a t-spanner path in the graph $\Theta(S, \kappa)$ between the points p and q. Is this path necessarily the shortest path in $\Theta(S, \kappa)$ between p and q?