Bottom-Up Parsing

COMP3002

School of Computer Science

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Bottom-Up Parsing

- We start with the leaves of the parse tree (individual tokens) and work our way up to the root.

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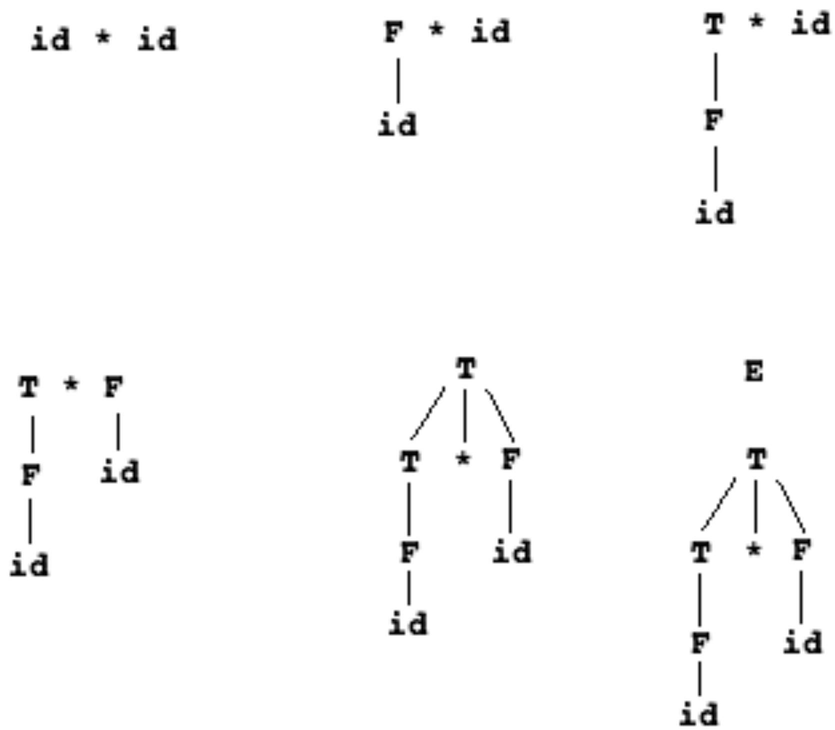
Example

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \textbf{id}$$



Reductions

- Find a **handle**
  - Elements in the string that form the right-hand side of a production in the grammar.
- Replace
  - Replace the handle with the left-hand side of the grammar it matches.
- Stop
  - When we end up with only the start symbol, we stop.
- Derivation in reverse
  - Reverse the reductions to get a sequence of derivations.
  - Specifically, a rightmost derivation.

Example (again…)

- We constructed the tree using 5 reductions…

id * **id**

F * **id**

T * **id**

T * *F*

F

E

$$
\begin{array}{l}
E \rightarrow E + T \mid T \\
T \rightarrow T * F \mid F \\
F \rightarrow (E) \mid \mathbf{id}
\end{array}
$$

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Shift-Reduce Parsing

- Shift
  - Shift the next element on the input to the top of the stack.
- Reduce
  - There is a handle on top of the stack. Replace those elements with the left-hand side of the associated production.
- Accept
  - There is no more input to process.
  - The stack consists only of the start symbol.
- Error
  - Syntax error discovered.

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Example (yet again!)

$$
\begin{aligned}
E &\rightarrow E + T \mid T \\
T &\rightarrow T * F \mid F \\
F &\rightarrow (E) \mid \mathbf{id}
\end{aligned}
$$

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Shift-Reduce Conflict

- Can't decide whether to shift or reduce…

stmt	→	**if** *expr* **then** *stmt*
	|	**if** *expr* **then** *stmt* **else** *stmt*
	|	**other**

- Consider following stack configuration…

STACK	INPUT
$ … **if** *expr* **then** *stmt*	**else** … $

Reduce-Reduce Conflict

- What production to reduce by?

$$
\begin{array}{ccc}
 & \ldots & \\
E & \rightarrow & S \\
F & \rightarrow & S \\
 & \ldots &
\end{array}
$$

- What production to reduce by?

STACK	**INPUT**
$ … **S**	… $

LR Parsers

- LR(k) Parsers
  - L for left-to-right scanning
  - R for rightmost derivation
  - k symbols of lookahead
- Different types…
  - Simple LR (SLR)
  - Canonical-LR
  - LALR

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Why is this good?

- Lookahead is easier
  - LR(k) looks ahead *k* symbols in a right-sentential form, and matches a production.
  - LL(k) tries to recognize a production from the first *k* characters of the string it derives.
  - So, more grammars.
- Error Handling
  - Detect syntax errors as soon as they occur.

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Conflict Resolution

- Construct a finite automaton (FA) that recognizes the right-hand-side of productions by scanning the input from **right to left**.
- Items
  - An item of G is a production of G with a dot at some position of the body.
  - Eg, $A \rightarrow X\,.\,YZ$
  - A state in our FA is a set of items.
- This is ***Simple LR (SLR) Parsing***

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Break

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Constructing the Finite Automaton

We need the Canonical Collection of LR(0) Items.

1. Augment the grammar
2. CLOSURE of items
3. GOTO function between items

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Augmented Grammars

- To augment grammar G with start symbol S, we add a new production $S' \rightarrow S$ and make S' the new start symbol.

E’	→	E
E	→	E + T | T
T	→	T * F | F
F	→	(E) | **id**

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Closure

- CLOSURE(I), where I is a set of items for a grammar G.
  1. Initially, add every item in I to CLOSURE(I)
  2. If $A \rightarrow x\ .\ B\ y$ is in CLOSURE(I), and $B \rightarrow .\ w$ is a production, add $B \rightarrow .w$ to CLOSURE(I) if it isn't there already.
  3. Apply Rule 2 until no more new items are added to CLOSURE(I).

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Example

- If $I = \{ E' \rightarrow .E \}$, then CLOSURE(I):

$E' \rightarrow E$
$E \rightarrow .\, E + T$
$E \rightarrow .\, T$
$T \rightarrow .T * F$
$T \rightarrow .\, F$
$F \rightarrow .(\, E\,)$
$F \rightarrow .\, \textbf{id}$

E'	→	E
E	→	E + T | T
T	→	T * F | F
F	→	(E) | **id**

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The GOTO Function

- *GOTO(I, X)* defined where *I* is an item and *X* is a grammar symbol.
- Defines the transitions between sets of **items** in the finite automaton.
- If [$A \rightarrow \mathbf{a} \,.\, X\, \mathbf{b}$] is in I, *GOTO(I, X)* contains CLOSURE($A \rightarrow \mathbf{a}\, X \,.\, \mathbf{b}$)

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Example

I

$E' \rightarrow E\,.$

$E \rightarrow E\,.+T$

$GOTO(I, +)$

$E \rightarrow E + .\,T$

$T \rightarrow .\,T * F$

$T \rightarrow .\,F$

$F \rightarrow .\,(E)$

$F \rightarrow .\,\mathbf{id}$

E'	→	E
E	→	E + T | T
T	→	T * F | F
F	→	(E) | **id**

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A fat groundhog

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Canonical Collection of LR(0) items

```
C = CLOSURE( { S' → . S } )
repeat
    for each set of items I in C
        for each grammar symbol X
            if GOTO(I,X) is not empty and not in C
                add GOTO(I,X) to C
until no new sets of items are added to see
```

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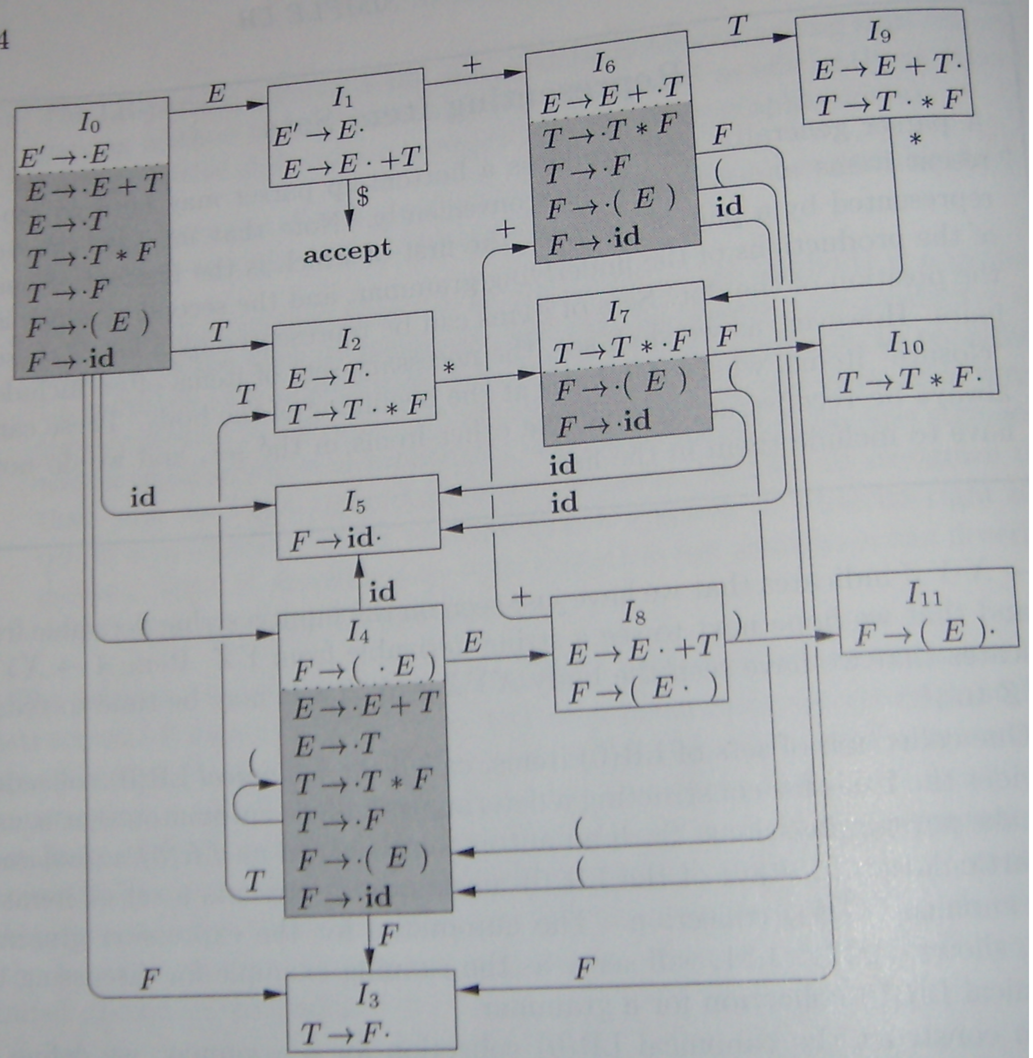

Figure 4.31: LR(0) automaton for the expression grammar (4.1)

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Simple LR Parsing

- Parse Table
  - ACTION and GOTO functions
  - Built from the finite automaton
- GOTO
  - Defined as before
- ACTION
  - If $[A \rightarrow A\ .\ \mathbf{x}\ B\ ]$ is in I_i, and GOTO(I_i, x) = I_j, then
    - ACTION(i, **x**) = "shift j"
  - If $[A \rightarrow X\ .\ ]$ is in I_i, then
    - ACTION(i, a) = "reduce A →X" for all a in FOLLOW(A).

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Elements of SLR Parser

- Stack
  - Maintains a stack of **states**
  - Used to resolve conflicts.
- Symbols
  - Grammar symbols corresponding to states on the stack

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Shift-Reduce Parsing

- ACTION[s,a] = shift j
  - Push j onto the stack
  - Append a to the input symbols
- ACTION[s,a] = reduce $A \rightarrow X$
  - Pop |X| symbols off the stack
  - Let t be state on top of the stack
  - Push GOTO[t, A] onto the stack
- Accept, Error
  - As before

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Example

STATE	action						goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Fig. 4.31. Parsing table for expression grammar.

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Canonical LR Parsing

- In SLR, we always reduce by [$A \rightarrow B$.] on input a if it is in *FOLLOW(A)*.
- However, there may be some prefix *XYZA* that can never be followed by a.

- In Canonical LR Parsing, for each **item** we store a **lookahead** that we have to see before reducing. Eg, [$A \rightarrow B$. (a)]

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LALR

- Lookahead LR
- Canonical LR tables are typically an order of magnitude larger than SLR tables.
- Construct Canonical LR table, prune them.

Final Thoughts

- Hard to implement
  - Compared to LL(k) parsers.
  - In practice, don't construct them.
  - Instead, use parser generators.
- More powerful
  - Every LL(k) grammar is LR(k)
  - Reverse not necessarily true.

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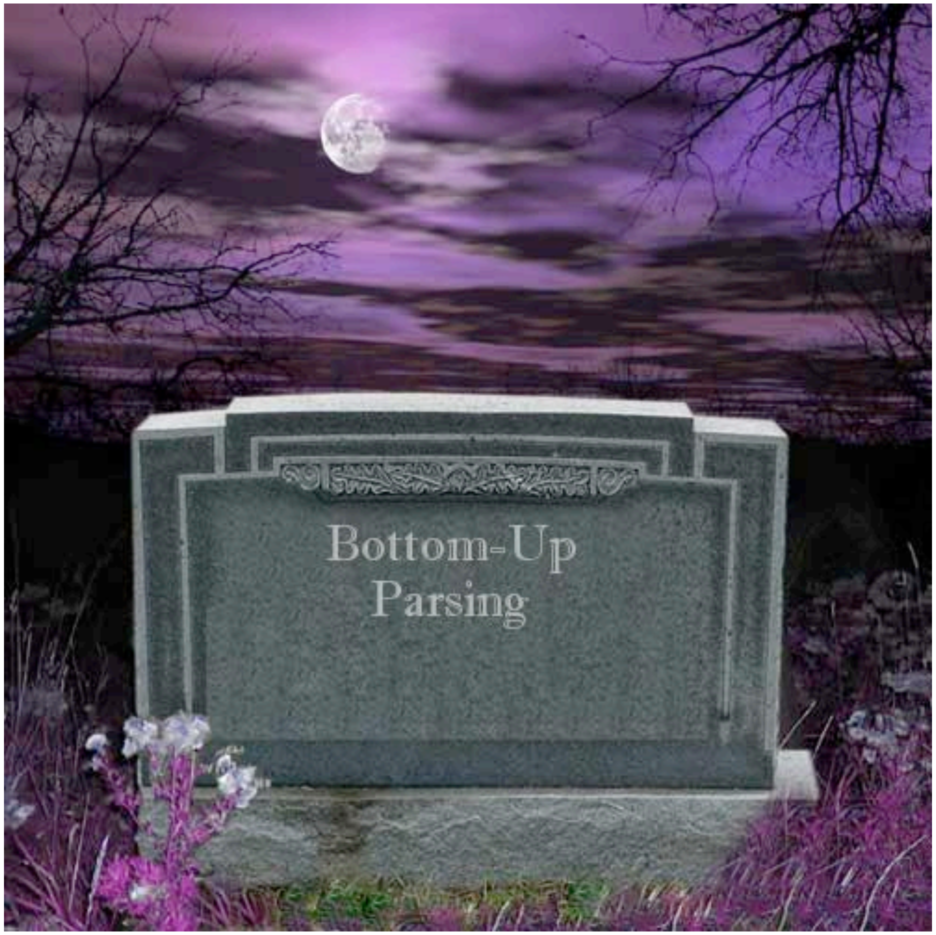
Fin!



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