COMP 3804 — Tutorial April's Fool Day, 2026

Problem 1: Let $x_1, x_2, \ldots, x_n$ be Boolean variables, and let φ be a Boolean formula of the form

$$\varphi = C_1 \wedge C_2 \wedge \ldots \wedge C_m,$$

where each clause C_i, $1 \leq i \leq m$, is of form

$$C_i = l_1^i \vee l_2^i \vee l_3^i.$$

Each l_j^i is a *literal*, which is either a variable or the negation of a variable. Note that each clause has exactly three literals.

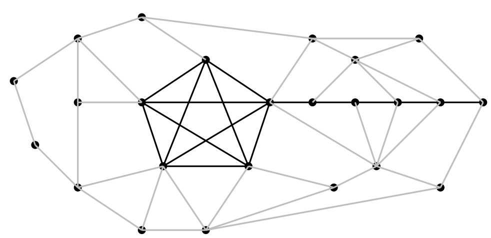
Assume that in each clause, its literals involve three distinct variables. For example, $x_1 \vee \neg x_2 \vee x_3$ is a valid clause, whereas $x_1 \vee \neg x_1 \vee x_2$ is not valid.

Prove the following: If the number m of clauses is at most 7, then φ is satisfiable.
Hint: Give each variable x_j a uniformly random truth-value. Define the random variable X whose value is the number of clauses that are true. Prove that $\mathbb{E}(X) = 7m/8$.

Problem 2: The *clique problem* is defined as follows:

$$\textsc{Clique} = \{(G, K) : \text{ graph } G \text{ contains a clique of size } K\}.$$

A K-*kite* is a graph consisting of a clique of size K and a path with K vertices that is connected to one vertex of the clique; thus, the number of vertices is equal to $2K$. In the figure below, the graph with the black edges forms a 5-kite.



The *kite problem* is defined as follows:

$$\textsc{Kite} = \{(G, K) : \text{ graph } G \text{ contains a } K\text{-kite}\}.$$

Prove that $\textsc{Clique} \leq_P \textsc{Kite}$, i.e., in polynomial time, $\textsc{Clique}$ can be reduced to $\textsc{Kite}$.

Problem 3: The *subset sum problem* is defined as follows:

$$\textsc{SubsetSum} = \{(S, t) : \begin{array}{l} S \text{ is a set of integers, } t \text{ is an integer,} \\ \exists S' \subseteq S \text{ such that } \sum_{x \in S'} x = t \ \}. \end{array}$$

The *partition problem* is defined as follows:

$$\textsc{Partition} = \{S : \begin{array}{l} S \text{ is a set of integers,} \\ \exists S' \subseteq S \text{ such that } \sum_{x \in S'} x = \sum_{y \in S \setminus S'} y \ \}. \end{array}$$

- Prove that SubsetSum $\leq_P$ Partition, i.e., in polynomial time, SubsetSum can be reduced to Partition.
- Prove that Partition $\leq_P$ SubsetSum, i.e., in polynomial time, Partition can be reduced to SubsetSum.

Problem 4: The *clique and independent set problem* is defined as follows:

$$\text{CliqueIndepSet} = \{(G,K) : \begin{array}{l} \text{graph } G \text{ contains a clique of size } K \text{ and} \\ G \text{ contains an independent set of size } K \ \}. \end{array}$$

Prove that Clique $\leq_P$ CliqueIndepSet, i.e., in polynomial time, Clique can be reduced to CliqueIndepSet.

Problem 5: Let φ be a Boolean formula in the variables $x_1, x_2, \ldots, x_n$. We say that φ is in *conjunctive normal form* (CNF) if it is of the form

$$\varphi = C_1 \wedge C_2 \wedge \ldots \wedge C_m,$$

where each C_i, $1 \leq i \leq m$, is of the following form:

$$C_i = l_1^i \vee l_2^i \vee \ldots \vee l_{k_i}^i.$$

Each l_j^i is a *literal*, which is either a variable or the negation of a variable.

The *satisfiability problem* is defined as follows:

$$\text{Sat} = \{\varphi : \varphi \text{ is in CNF-form and is satisfiable}\}.$$

Prove that Clique $\leq_P$ Sat, i.e., in polynomial time, Clique can be reduced to Sat.